Negotiating Parenthood: An Account of the Changing Relationship between Women's Work and Fertility*

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Abstract

In high-income economies, the cross-country relationship between women's labor force participation and fertility has reversed in recent decades: countries where many women work used to have fewer children, but now they have more. In this paper, we examine the role of bargaining frictions in the household in accounting for this reversal. We develop a model in which partners bargain over how many children to have under limited commitment. Frictions arise because having children changes outside options. Bargaining frictions are large, and fertility is low, if mothers provide the majority of childcare while giving up lucrative career opportunities to raise children. In a simple setting, the bargaining friction is proportional to the marginal child penalty. When the only alternative for women is to raise children or to work, the relationship between women's labor supply and fertility is negative. When women's labor market opportunities are sufficiently attractive that market childcare is used, the relationship reverses. The model can account for the broad patterns in women's labor supply, child penalties, and fertility observed in high-income countries in recent decades.

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1 Introduction

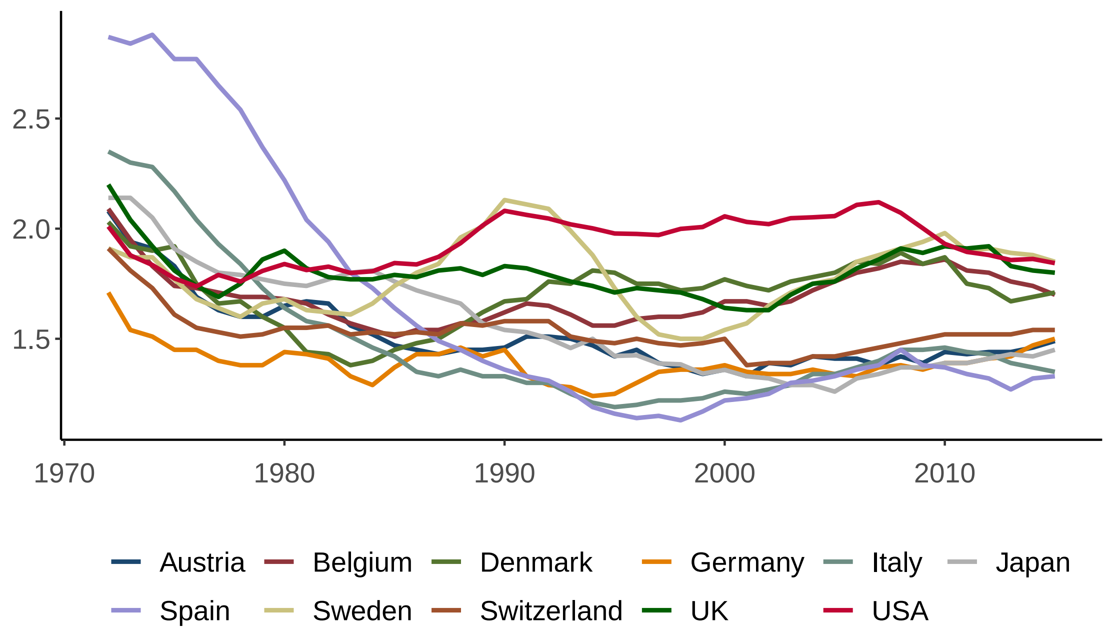
From the 1960s to the present, families in high-income countries have gone through a remarkable transformation. At the beginning of this period, the traditional single-earner family was still dominant; the labor force participation rate of married women was low, and conversely married men did little childcare or housework. Since then, married women's labor force participation has risen substantially, and is close to that of men in many countries. Time use data show that while women continue to provide the majority of childcare and housework, men's contributions to this task have increased greatly over the same time period.

Basic economic models of fertility would suggest that such a transformation in the division of labor in the family should lead to a substantial decline in fertility rates. The increase of mothers' labor force participation implies a rise in the opportunity cost of raising children, and if more childcare is done by men (who have higher wages on average) that would increase the cost of children even more. Consistent with this idea, at the beginning of the period under consideration there was a clear negative relationship between women's labor force participation and fertility rates at a country level.

Nevertheless, the evolution of fertility rates since the 1960s contradicts the notion of a simple, monotonic relationship between women's labor force participation and fertility. First, as shown in Figure 1, fertility rates have been broadly stable in high-income countries since the 1980s, even though women's labor force participation has continued to increase. Second, as shown in Figure 2, over the same period the cross-country relationship between women's labor force participation and fertility has reversed: from the late 1980s onwards, countries where more women work had higher fertility rates compared to countries where the traditional division of labor prevails.

The aim of this paper is to evaluate the role of intra-family bargaining in accounting for the evolving relationship of the division of labor in the family and fertility behavior. To this aim, building on Doepke and Kindermann (2019) we develop a model in which couples bargain over how many children to have. The two parents may have different preferences over having children and they may face different costs of childcare. Bargaining over fertility takes place under limited commitment, which implies that parents take into account how having children will affect their outside options and bargaining power in future intra-family bargaining. We show that in this setting there can be a "bargaining wedge" between the actual number of children a couple has versus the jointly efficient number of children that they would choose in the absence of bargaining frictions.

Figure 1: Total Fertility Rates since 1970



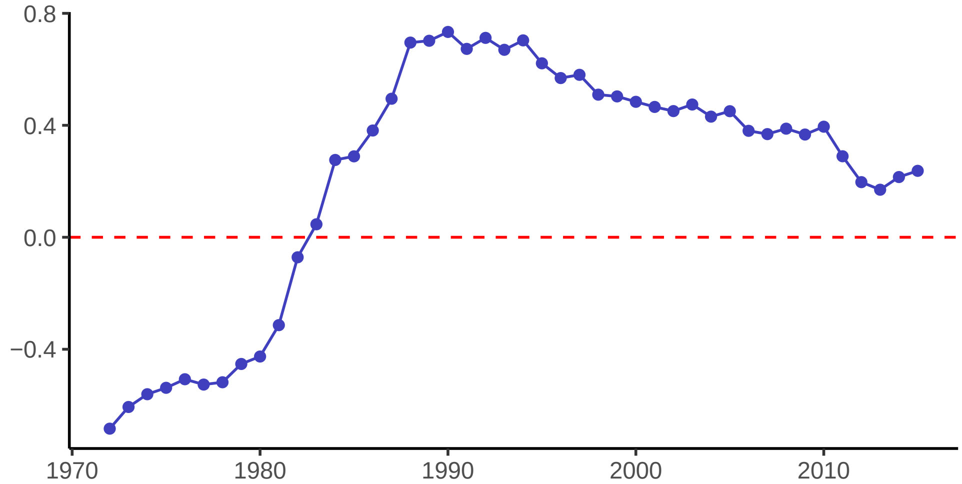
Source: Data from the World Bank World Development Indicators

A key insight of the paper is that the size of the bargaining wedge depends directly on the marginal child penalty, i.e., the loss in future earnings that a mother would experience if the couple were to choose one more child. The loss in future earnings through the marginal child penalty directly affects the mother's outside option, and thus implies an additional cost of having another child for the mother (and a corresponding benefit for the father).

We show that in our model, variation in the marginal child penalty over time and across countries can generate a changing relationship between women's labor supply and fertility that lines up with what is observed in the data. Importantly, the model generates a reversing relationship between women's labor supply and fertility as a result of a single driving force, namely gradually improving labor market conditions for mothers. What drives this finding is that a steady increase in labor market opportunities implies that the marginal child penalty follows an inverted-U pattern.

To understand this pattern, consider the case where women's changing labor market opportunities are driven by a steady rise in women's relative wages w_f, whereas men's wages w_m and the cost of childcare p are constant. Starting out from the setting where w_f is very low, mothers will provide childcare themselves given their low opportunity cost

Figure 2: The Changing Cross-Country Correlation between Women's Labor Force Participation Rate and the Total Fertility Rate



Source: Graph from Ahn and Mira (2002), own replication

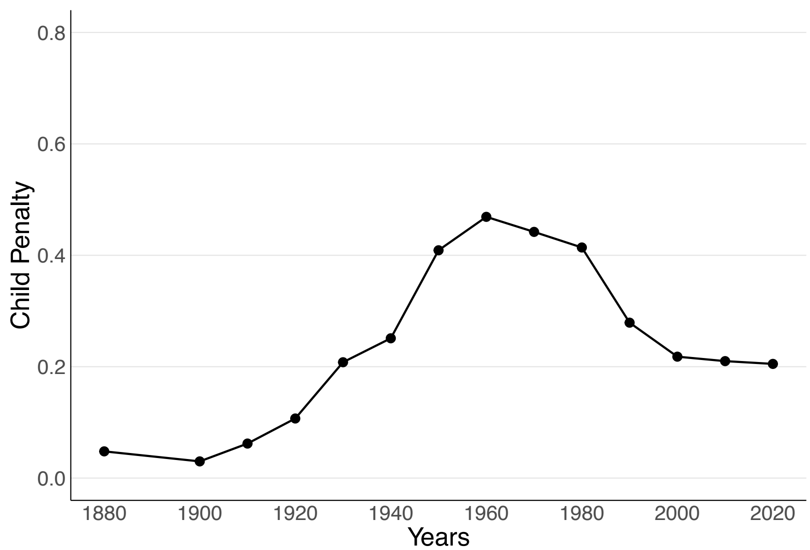
of time. Nevertheless, the marginal child penalty is also low, because given low wages, an increase in time spent on raising children does not result in a large reduction in earnings. Thus, we start out in a regime of low marginal child penalties. Empirically, this regime correspond to a period where both married women's labor force participation and married women's relative wages are low. In such a setting, having another child does little to change a woman's outside option, implying a small bargaining wedge.

Starting from this regime, if women's wages w_f start rising, the marginal child penalty will also start to increase, because the value of women's forgone earnings from having another child will rise with wages. This effect of rising wages will dominate as long as women continue to provide most of childcare themselves. Hence, with an intermediate level of wages, there will be a larger marginal child penalty and hence a larger bargaining wedge.

When women's wages continue to increase, a second effect comes into play, namely that an increase in w_f will induce women to provide less childcare themselves and rely more on market childcare. The two effects are competing: for given childcare time, an increase in w_f increases the marginal child penalty, but a decline in childcare time decreases it. Whether the first or the second effect dominates crucially depends on how elastic maternal childcare time is to the wage rate. If this elasticity is high enough then the second effect will ultimately come to dominate, and the marginal child penalty starts to fall. If wages rise sufficiently so that women always continue to work and hence do not experience any decline in future earnings from having another child, the marginal child penalty will disappear, just as in the opposite case where women are entirely unable to work.

Our findings are generally consistent with recent findings from the empirical literature on the child penalty, see for example Kleven, Landais, and Søgaard (2019) or Kleven, Landais, and Leite-Mariante (2024). In their recent work on the "Child Penalty Atlas", Kleven, Landais, and Leite-Mariante (2024) have estimated historical child penalties in the US, see Figure 3. Their data in fact show an explicit hump-shape of the child penalty over time.

Figure 3: Historical Child Penalties in the United States (1880-2020)



Source: Kleven, Landais, and Leite-Mariante (2024)

The remainder of this paper is organized as follows. In Section 2, we present a simple model of how bargaining matters for fertility outcomes. We then analyzes how a bargaining wedge induced by a lack of commitment in the family relates to the child penalty. Finally, we provide some comparative statics and an illustrative example. In Section 3, we demonstrate show to estimate marginal child penalties and present preliminary data from the German Socio-Economic Panel. The final section concludes.

2 A Simple Model

We illustrate the main ideas with a model that only includes the essential elements to generate a changing relationship between fertility and female labor force participation through bargaining frictions.

Let us consider an economy populated by couples composed of a woman and a man $g \in \{f, m\}$ who decide on individual consumption c_g and the number of children n. Preferences of an individual of gender g are given by:

$$u(c_g, n) = c_g + v(n),$$

where the utility from children $v(n)$ is increasing and concave. We impose linear utility from consumption to focus on the case of transferable utility between the spouses and assume that both partners have the same preference for children; we discuss how this can be generalized below.

Having n children comes at (time and goods) costs $d_g(w_f, w_m, p, n)$ for each partner g. These individual level costs depend on the wages w_g – reflecting individual time costs –, the price p of buying child care, and the number of children n. For now, we assume that the cost functions $d_g(\cdot)$ are given and cannot be influenced by the couple. This allows us to provide a general view on the determinants of the couple's decision problem. Later, we will offer a micro-foundation for how these costs emerge and how they may be split across partners. The total cost of raising n children for the couple consequently is

$$d(w_f, w_m, p, n) = d_f(w_f, w_m, p, n) + d_m(w_f, w_m, p, n).$$

We now formulate some assumptions towards the child cost functions that ensure that couples want to have a finite number of children and therefore keep the model tractable.

Assumption 1. *The child costs $d_g(w_f, w_m, p, n)$ are differentiable in n for all $n > 0$ and they satisfy:*

1. *no cost without children: $d_g(w_f, w_m, p, 0) = 0$*
2. *no children without cost: $d_g(w_f, w_m, p, n) > 0$ for all $n > 0$*
3. *more children, more cost: $\frac{\partial d_g(w_f, w_m, p, n)}{\partial n} > 0$ for all $n > 0$*

The household faces a budget constraint

$$c_f + c_m = (1+\alpha)\left[w_m + w_f - d(w_f, w_m, p, n)\right]. \tag{1}$$

Consumption is financed from total income $w_f + w_m$ minus the cost of having children. In the case the couple decides jointly on the consumption allocation, the household's resources are multiplied with a factor $\alpha > 0$ that reflects increasing returns from joint consumption in the family, for instance by sharing household public goods such as a house or apartment.

The couple takes decisions in two steps:

1. First, the couple has to decide on their fertility choice n. This decision is done through a veto model, where each partner is able to block a further increase in the fertility rate. This implies that the chosen fertility rate will be the minimum of the fertility rates preferred by each spouse.
2. Second, the couple decides on the allocation of consumption. This decision is taken under Nash bargaining with outside options that depend on each spouse's labor earnings as well as the prior fertility decision.

We now characterize these two decision steps starting with the consumption allocation. We then turn to how this allocation feeds back into the fertility decision in the first stage.

2.1 The consumption allocation

The couple bargains over how to allocate consumption using Nash bargaining with equal weights. Outside options in the bargaining game are determined by a situation in which increasing returns to consumption are forgone, each spouse relies on their own income, and each spouse finances their share of the cost of children $d_g(w_f, w_m, p, n)$. Furthermore, we assume that each partner is still able to enjoy utility from their children, rendering the outside option a state of non-cooperation rather than formal separation, see Lundberg and Pollak (1993). Utility under the outside option is

$$\bar{u}_g(w_f, w_m, p, n) = \underbrace{w_g - d_g(w_f, w_m, p, n)}_{=\bar{c}_g} + v(n).$$

Next to the individual wage w_g, the major determinant of utility in the outside option is the cost distribution of children $d_g(\cdot)$. The partner who takes the larger share in managing the cost of children faces a penalty on her or his consumption. To formalize this idea, let us define the bargaining loss or gain from the perspective of the female household member as

$$l(w_f, w_m, p, n) = d_f(w_f, w_m, p, n) - d_m(w_f, w_m, p, n).$$

$l(\cdot)$ indicates the loss in bargaining position for the wife relative to the husband from having children.

The Nash bargaining problem that determines the consumption allocation for a given number of children n can then be written as

$$\max_{c_f, c_m} \Big\{ \left[c_f + v(n) - \bar{u}_f(w_f, w_m, p, n)\right]\left[c_m + v(n) - \bar{u}_m(w_f, w_m, p, n)\right] \Big\},$$

where the maximization is subject to the budget constraint in (1). Solving this allocation is straightforward and leads to the following consumption for the mother:

$$c_f = \underbrace{w_f}_{\substack{\text{ex-ante}\\ \text{outside option}}} \underbrace{- \frac{l(w_f, w_m, p, n)}{2}}_{\text{bargaining loss}} \underbrace{- \frac{d(w_f, w_m, p, n)}{2}}_{\substack{\text{equal share}\\ \text{of child cost}}} + \underbrace{\frac{\alpha}{2}\big[w_m + w_f - d(w_f, w_m, p, n)\big]}_{\text{share of cons. surplus}}$$

In contrast, the father's consumption is:

$$c_m = \underbrace{w_m}_{\substack{\text{ex-ante}\\ \text{outside option}}} \underbrace{+ \frac{l(w_f, w_m, p, n)}{2}}_{\text{bargaining gain}} \underbrace{- \frac{d(w_f, w_m, p, n)}{2}}_{\substack{\text{equal share}\\ \text{of child cost}}} + \underbrace{\frac{\alpha}{2}\big[w_m + w_f - d(w_f, w_m, p, n)\big]}_{\text{share of cons. surplus}}$$

The consumption allocation between wife and husband first and foremost depends on the ex-ante outside option w_g, meaning the wage a partner would earn in a state of non-cooperation without children. If both partners were to share the costs of children equally, then having children would reduce each partner's consumption by $\frac{1+\alpha}{2}d(w_f, w_m, p, n)$. However, owing to the two-step sequencing of the bargaining game, in which outside options directly depend on prior fertility decisions, it is not only the total cost of children that matters for the consumption allocation but also its distribution across partners. Consistent with the idea phrased above, whenever the female partner faces a higher cost of having children, she experiences a loss in bargaining power and consequently her consumption declines by an additional $\frac{l(w_f, w_m, p, n)}{2}$. In turn, her husband's consumption increases by the same amount, creating a lopsided distribution of the cost of children within the family.

2.2 The fertility decision

With the consumption allocation at hand, we are able to determine the couple's optimal fertility decision. Without loss of generally, let us assume that $l(w_f, w_m, p, n) \geq 0$.[1] A positive bargaining loss for women implies that with identical direct utility from having children $v(n)$, the mother prefers to have fewer because her effective cost of having children is higher. Given the veto model of decisions on children, the mother's desire to have children will therefore be pivotal in the fertility decision.

Let us first investigate the intensive margin fertility choice, i.e. the optimal number of children provided the couple decides to have any. Given that the mother's utility reads

$$u_f = w_f - \frac{l(w_f, w_m, p, n)}{2} - \frac{d(w_f, w_m, p, n)}{2} + \frac{\alpha}{2}\big[w_m + w_f - d(w_f, w_m, p, n)\big] + v(n),$$

the optimal number of children is characterized by

$$v'(n^\star) = \frac{1}{2}\Bigg[\underbrace{\frac{\partial l(w_f, w_m, p, n^\star)}{\partial n}}_{\text{marginal bargaining loss}} + \underbrace{(1+\alpha)\frac{\partial d(w_f, w_m, p, n^\star)}{\partial n}}_{\text{marginal effective child cost}}\Bigg].$$

The optimum number of children depends on both how the total cost of children changes with n and how the distribution of the cost of children – as measured by the mother's bargaining loss – evolves with n. Fertility can therefore be low because the cost of children is generally high, or because mother's have to take a larger share in the cost of having (more) children.

Depending on the exact shape of the child-cost function $d_g(\cdot)$ and the fertility preference $v(n)$ around the value $n = 0$, the couple may also have to decide whether they want to have children at all. Along this extensive margin, women have to compare a situation in which they decide to have no children with the situation of having $n^\star$ children. Women will agree to having children whenever

$$v(n^\star) - v(0) \geq \frac{1}{2}\Big[\underbrace{l(w_f, w_m, p, n^\star)}_{\text{absolute bargaining loss}} + \underbrace{(1+\alpha)d(w_f, w_m, p, n^\star)}_{\text{absolute effective child cost}}\Big].$$

[1] This assumption is made purely to avoid case differentiation. In addition, the case where mothers take a larger share of the cost of having children also seems to be the empirically more relevant case, see for example (Kleven et al. 2019).

The extensive margin decision has a very similar form as the intensive margin decision. However, there is a clear distinction to be made between marginal and absolute costs. The marginal cost and bargaining loss indicate how the situation changes for the mother as she will have more children. Hence, these statistics determine the optimal quantity of children. The absolute costs and bargaining losses, on the other hand, indicate whether mothers are at all willing to have children.

The role of commitment Before we proceed with providing a micro-founded model for both the cost of children and its distribution, let us quickly point to the role commitment plays in determining the prior results. Let us for a moment assume that the couple were able to perfectly commit to sharing the full cost of having children equally. Then the optimality conditions for fertility would read

$$v'(n^\star) = \frac{1}{2}\underbrace{(1+\alpha)\frac{\partial d(w_f, w_m, p, n^\star)}{\partial n}}_{\text{marginal effective child cost}} \quad \text{and} \quad v(n^\star) - v(0) \geq \frac{1}{2}\underbrace{(1+\alpha)d(w_f, w_m, p, n^\star)}_{\text{absolute effective child cost}}. \tag{2}$$

for the intensive and the extensive margin, respectively. This means that without bargaining frictions, the only statistic that was relevant in determining fertility would be the total cost of children and its evolution with the number of children. The distribution of costs among partners would play no role.

2.3 A micro-foundation for costs and bargaining losses

We now want to characterize how the cost of having children $d(\cdot)$ as well as the bargaining loss $l(\cdot)$ depend on observables. Children typically come with both goods costs $\psi(n)$ and time costs $\phi(n)$. We require that these costs increase in the number of children, i.e. $\psi'(n) > 0$ and $\phi'(n) > 0$.

Time costs can be borne either directly by the mother t_f at price w_f or by buying child care τ at price p on the market.[2] Child care provided by parents and through child-care facilities may not be perfect substitutes. We therefore let the time requirement to raise

[2] Time costs could of course also be borne by the father. We consider an extension that allows for fathers' participation in child care below. For the clearness of our argument, we focus on women's child care time for now. Note that empirically, the average woman takes a much larger share in caring for children in many countries, see for example Kleven et al. (2019).

children satisfy

$$F(t_f,\tau)=\phi(n) \text{ with } \frac{\partial F(\cdot)}{\partial t_f},\frac{\partial F(\cdot)}{\partial \tau}>0,\ \frac{\partial^2 F(\cdot)}{\partial t_f^2},\frac{\partial^2 F(\cdot)}{\partial \tau^2}\le 0 \text{ and } \lim_{t_f\to 0^+}\frac{\partial F(t_f,\tau)}{\partial t_f}=\infty.$$

The last condition ensures that child-care activities cannot be outsourced entirely to the market. The couples chooses the childcare arrangement that minimizes total costs. Consequently, the total cost of raising children reads

$$d(w_f,w_m,p,n)=\psi(n)+\min_{t_f,\tau}\ w_f t_f+p\tau \quad \text{s.t} \quad F(t_f,\tau)=\phi(n).$$

The cost-minimal solution is characterized by

$$\frac{\partial F(\cdot)/\partial t_f}{\partial F(\cdot)/\partial \tau}\ge \frac{w_f}{p}$$

where this equation holds with equality whenever $\tau>0$.

Let us assume that the couple can agree on sharing all monetary costs of children equally, meaning both the goods cost $\psi(n)$ and the cost of buying child care $p\tau$. Let use further assume that the child care arrangement made by the couple is binding even in a state of non-cooperation. Consequently, whenever the mother provides child care by herself, this lowers her outside option in the bargaining game. We can write

$$d_f(w_f,w_m,p,n)=w_f t_f(w_f,w_m,p,n)+0.5\big[p\tau(w_f,w_m,p,n)+\psi(n)\big] \quad \text{and}$$
$$d_m(w_f,w_m,p,n)=0.5\big[p\tau(w_f,w_m,p,n)+\psi(n)\big],$$

where $t_f(w_f,w_m,p,n)$ and $\tau(w_f,w_m,p,n)$ denote the solutions to the child-care organization problem. The bargaining loss then immediately reads

$$l(w_f,w_m,p,n)=w_f t_f(w_f,w_m,p,n). \tag{3}$$

The bargaining loss arises from the child-care duties of the mother that lower her outside option and hence weaken her bargaining power in the consumption allocation bargaining game. Note that $w_f t_f(w_f,w_m,p,n)$ denotes the income loss of a mother after child birth, which the empirical literature calls the *child penalty*.

Recall that the absolute level of child costs and child penalty are only relevant in determining whether a woman wants to have any children or not. In order to calculate the optimum quantity of children, we are interested in *marginal child costs* and particularly in the *marginal child penalty*. Note that for a given set of wages w_f, w_m and a price of child care p, we have

$$\frac{\partial l(w_f, w_m, p, n)}{\partial n} = w_f \frac{\partial t_f(w_f, w_m, p, n)}{\partial n} \quad \text{and} \quad \frac{\partial d(w_f, w_m, p, n)}{\partial n} = \psi'(n) + \frac{w_f \phi'(n)}{\partial F(\cdot)/\partial t_f}.$$

2.4 Comparative statics

The model formulated above is quite general and it will therefore be hard to provide any further analytical solutions. In order to keep the model tractable and to be able to conduct a comparative statics analysis, let use make the following assumptions towards the child-care technology:

Assumption 2. *The child care costs are $\psi(n) = \psi n$ and $\phi(n) = \phi n$ and the child care technology satisfies*

$$F(t_f, \tau) = f\left(\tilde{t}_f, \tilde{\tau}\right) \times n \quad \textit{with} \quad \tilde{t}_f = \frac{t_f}{n} \ \textit{and} \ \tilde{\tau} = \frac{\tau}{n}.$$

With these assumptions, we can write the total child care cost as

$$d(w_f, w_m, p, n) = n \times \underbrace{\left[\psi + \min_{t_f, \tau} \ w_f \tilde{t}_f + p\tilde{\tau}\right]}_{=\tilde{d}(w_f, w_m, p)} \quad \text{s.t} \quad f(\tilde{t}_f, \tilde{\tau}) = \phi$$

and the child penalty is

$$l(w_f, w_m, p, n) = n \times w_f \tilde{t}_f(w_f, w_m, p). \tag{4}$$

The optimum quantity of children is therefore given by

$$v'(n^\star) = \frac{1}{2}\Big[\underbrace{w_f \tilde{t}_f(w_f, w_m, p)}_{\text{marginal child penalty}} + \underbrace{(1+\alpha)\tilde{d}(w_f, w_m, p)}_{\text{marginal effective child cost}}\Big].$$

Note that by the assumptions made in Assumption 2 child costs are proportional in the number of children n. Since curvature of the utility function $v(n)$ implies $\frac{v(n^\star)}{n^\star} > v'(n^\star)$, the female partner always agrees to having children and the intensive margin is be the only relevant decision.

We are now interested in the question of how fertility relates to female wages w_f.

Proposition 1. *The elasticity of optimal fertility $n^\star$ with respect to the woman's wage rate w_f is*

$$\varepsilon_{n^\star,w_f} = \frac{dn^\star}{dw_f}\frac{w_f}{n^\star} = \frac{2+\alpha+\varepsilon_{\tilde{t}_f,w_f}}{2v''(n^\star)n^\star} \times w_f\tilde{t}_f(w_f,w_m,p) \lesseqgtr 0. \tag{5}$$

Proof: see Appendix A. □

There are two channels through which female wages influence the fertility decision $n^\star$. Assume for a moment that $\varepsilon_{\tilde{t}_f,w_f} = 0$, meaning that women's child care time is fixed per child. Then we unanimously have $\varepsilon_{n^\star,w_f} < 0$ and the relevant statistic for determining the fertility rate $n^\star$ is the size of marginal child penalty $w_f\tilde{t}_f(w_f,w_m,p)$. With a constant child care time, an increase in the woman's wage always raises the child penalty and therefore her optimal fertility level declines. The situation is different when a woman's child care time can react to changes in wages. When $\varepsilon_{\tilde{t}_f,w_f} < 0$, a rising wage rate for women leads the couple to buy more child care on the market. In this case, the child penalty may shrink with a rising wage rate, leading fertility to increase in w_f.

We now want to formalize these thoughts by making the following assumptions:

Assumption 3. *Let there be some threshold value $\bar{\omega} > 0$ such that $\varepsilon_{\tilde{t}_f,w_f} = 0$ for all $w_f < \bar{\omega}$ and $\varepsilon_{\tilde{t}_f,w_f} < 0$ for all $w_f \geq \bar{\omega}$. In addition, let*

$$\lim_{w_f\to\infty} \varepsilon_{\tilde{t}_f,w_f} = -\sigma \quad \textit{with} \quad \sigma > 1.$$

These assumptions state that for very low level's of wages w_f, the couple decides that all child care should be done by the mother and no child care is bought on the market. Only as the woman's wage passes a certain threshold $\bar{\omega}$ will the couple decide to complement female child caring time with time bought from outside the family. The second part of Assumption 3 states that we require the elasticity with which the child care time of women reacts to the wage rate to be finite and smaller than -1 as the wage rate increases to infinity.

The assumptions made above allow us to make some general statements on how the fertility rate changes along the female wage distribution.

Proposition 2. *For each $0 < w_f < \bar{\omega}$ we have $\varepsilon_{n^\star,w_f} < 0$. In addition, we have*

$$\lim_{w_f\to\infty} w_f\tilde{t}_f(w_f,w_m,p) = 0 \quad \textit{and} \quad \lim_{w_f\to\infty} \varepsilon_{n^\star,w_f} = 0.$$

Proof: see Appendix A. □

Proposition 2 states that fertility will be downward sloping for low levels of female wages w_f and that, as women's wages rise, fertility converges to some constant value. While the first property of the fertility rate appears quite natural – we have seen fertility rates decline along a transition that has seen rising wages and labor force participation for women – the second result is less immediate but more important. In fact, one could imagine the fertility rate to decline continuously with the wage rate for women so that it converges to zero as wages rise further and further. This would be the case if the couple's child caring decision would be entirely insensitive to the woman's wage w_f. However, we require in Assumption 3 that the woman's time spent at home with children does react to changes in wages, at least as the female wage rate is high enough. And importantly, it does so with an elasticity greater than 1 in absolute terms. Because of this, the woman's marginal child penalty declines to zero as w_f increases to infinity and all the cost of children will become entirely monetized. This feature prevents the fertility rate to decline all the way down to zero.

2.5 Female labor force participation and fertility

The results presented above suggest that the relationship between female labor activity and fertility in our model is all but clear cut. To put this on a more formal level, let us define the woman's labor supply as

$$\ell(w_f, w_m, p, n) = 1 - n\tilde{t}_f(w_f, w_m, p).$$

Proposition 3. *As a woman's wage increases, the relationship between her fertility rate and labor activity is*

$$\varepsilon_{\ell,n^\star} = -\frac{n^\star(w_f, w_m, p)\tilde{t}_f(w_f, w_m, p)}{\ell(w_f, w_m, p, n^\star)} \times \left[1 + \frac{\varepsilon_{\tilde{t}_f, w_f}}{\varepsilon_{n^\star, w_f}}\right]$$

Proof: see Appendix A. □

Proposition 3 shows that our model has the potential to generate a non-linear relationship between female labor force participation and fertility. Note that by Assumption 3, we have $\varepsilon_{\tilde{t}_f, w_f} = 0$ for all $w_f < \bar{\omega}$. In this region we therefore get a unanimously negative relationship between fertility and female labor supply, i.e. $\varepsilon_{\ell,n^\star} < 0$. The reason is obvious. Since women have to do all the work in caring for children, a rising wage rate increases her child penalty and therefore her cost of raising kids. With rising cost of children, a woman's desired optimal fertility rate declines. This frees up time that she would have otherwise used to care for children such that female labor supply increases. Consequently, in this region, we will see a negative relationship between fertility and female labor supply.

The situation may change when we enter a region in which $\varepsilon_{\tilde{t}_f,w_f} < 0$. In this case, whether fertility correlates positively or negatively with female labor supply fundamentally depends on the size of $\varepsilon_{\tilde{t}_f,w_f}$. In particular, when $\varepsilon_{\tilde{t}_f,w_f}$ is small in absolute terms then most likely we will see $\varepsilon_{n^\star,w_f} < 0$, too. The relationship between fertility and female labor supply will therefore remain negative. If, however, $\varepsilon_{\tilde{t}_f,w_f}$ is sufficiently large in absolute terms, then $\varepsilon_{n^\star,w_f}$ will turn positive and we would see a positive relationship between fertility and female labor supply. Summing up, the degree to which families decide to substitute child care time for mother's care time will be vital in understanding the relationship between fertility and labor supply along a fertility transition that is induced by rising female labor market opportunities. At the same time, $\varepsilon_{\tilde{t}_f,w_f}$ is a strong indicator for how the (marginal) child penalty evolves along such a fertility transition. Whenever $\varepsilon_{\tilde{t}_f,w_f}$ is large enough in absolute terms, the child penalty will decline quickly as female labor market opportunities increase and this will lead the economy to a path of rising female employment and fertility.

2.6 An example

In order to summarize and illustrate the previous results, let us look at the following example:

$$f(\tilde{t}_f, \tilde{\tau}) = \left\{\kappa(\tilde{t}_f)^{1-\frac{1}{\sigma}} + (1-\kappa)(\tau_0 + \tilde{\tau})^{1-\frac{1}{\sigma}}\right\}^{\frac{1}{1-\frac{1}{\sigma}}} \quad \text{with} \quad \tau_0 > 0 \text{ and } \sigma > 1.$$

The functional form proposed above is a specification with constant elasticity of substitution σ between mother's child-care time and child care bought on the market. Maternal child care time is a necessity. Child care bought on the market, however, doesn't have to be used by the couple. It rather serves as a "luxury good" in the sense that it will only be bought by couples for which the maternal wage exceed a certain threshold. The parameter τ_0 governs the extent to which market child care is a luxury good.

Under the above specification of the child care technology, the optimal choice for maternal child care is

$$t_f(w_f, w_m, p) = \begin{cases} \kappa^{\frac{\sigma}{1-\sigma}} \left[\phi^{\frac{\sigma-1}{\sigma}} - (1-\kappa)\tau_0^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} & \text{if } w_f < \bar{w} \text{ and} \\ [\kappa p]^\sigma \left[\kappa^\sigma p^{\sigma-1} + (1-\kappa)^\sigma (w_f)^{\sigma-1}\right]^{\frac{\sigma}{1-\sigma}} \phi & \text{otherwise,} \end{cases}$$

and the corresponding wage threshold is

$$\bar{w} = p \left[\frac{1-\kappa}{\kappa}\right]^{\frac{\sigma}{1-\sigma}} \left[(1-\kappa)^{-1} \left[\frac{\phi}{\tau_0}\right]^{\frac{\sigma-1}{\sigma}} - 1\right]^{\frac{1}{1-\sigma}}.$$

In turn, child care bought on the market depends on prices as follows:

$$\tau(w_f, w_m, p) = \begin{cases} 0 & \text{if } w_f < \bar{w} \text{ and} \\ [(1-\kappa)w_f]^\sigma \left[\kappa^\sigma p^{\sigma-1} + (1-\kappa)^\sigma (w_f)^{\sigma-1}\right]^{\frac{\sigma}{1-\sigma}} \phi - \tau_0 & \text{otherwise.} \end{cases}$$

Note that for this type of child care production function, $\tilde{t}_f(w_f, w_m, p)$ exactly fulfills the requirements specified in Assumption 3. This can be easily seen by calculating the elasticity

$$\varepsilon_{\tilde{t}_w, w_f} = \begin{cases} 0 & \text{if } w_f < \bar{w} \text{ and} \\ -\sigma \left[1 + \left(\frac{\kappa}{1-\kappa}\right)^\sigma \left(\frac{p}{w_f}\right)^{\sigma-1}\right]^{-1} & \text{otherwise.} \end{cases}$$

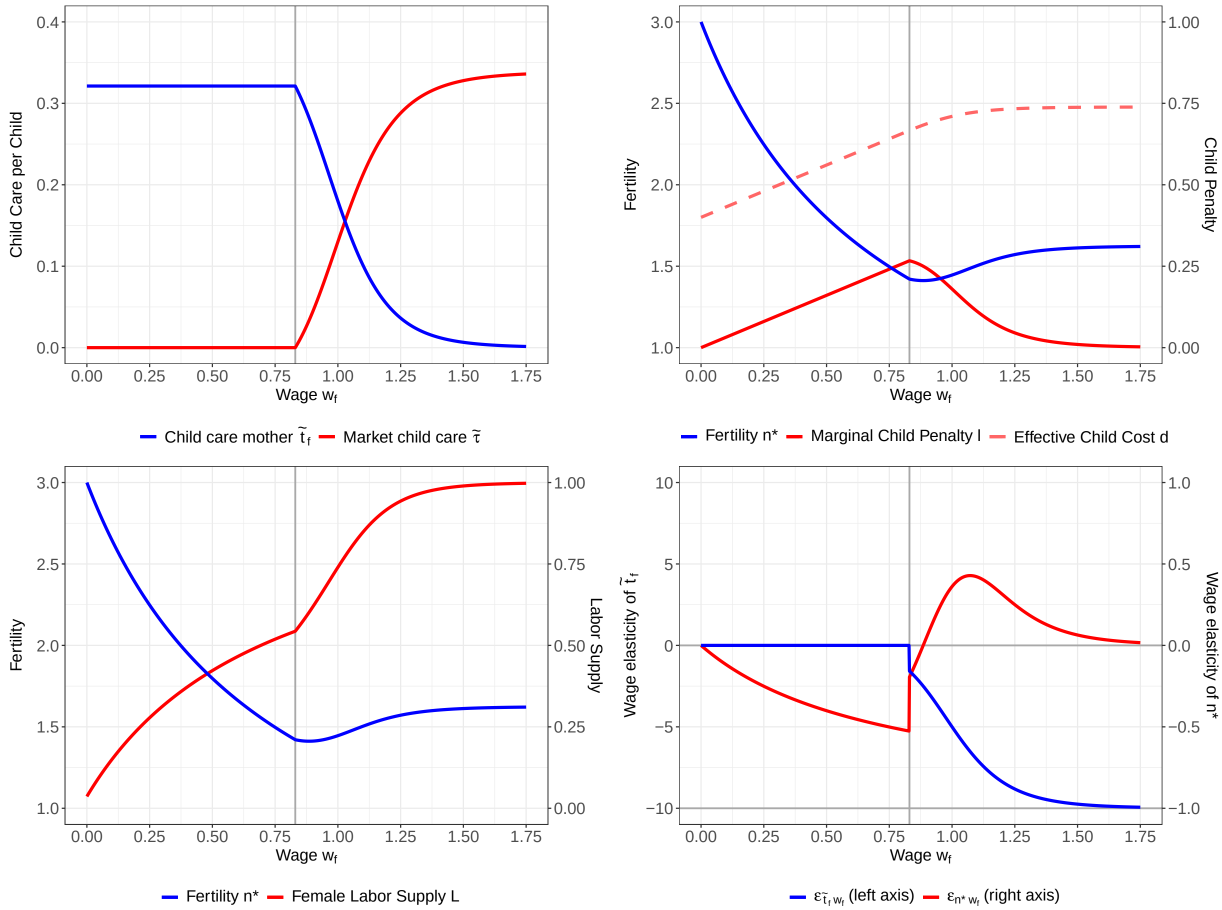
Finally, let us specify preferences for having children as $v(n) = \nu \log(n)$. Consequently, the fertility decision of the couple is

$$n^\star = \frac{2\nu}{\tilde{l}(w_f, w_m, p) + (1+\alpha)\tilde{d}(w_f, w_m, p)}.$$

Figure 4 illustrates the relationship between child care, fertility, the child penalty, female labor supply and women's wages in a parameterized version of the model. The horizontal axis in all four panels shows the mother's wage w_f. Note that we normalized the price of child care to $p = 1$. Hence, the woman's wage rate can be interpreted as her wage relative to the cost of child care.

The top left panel shows the couple's child care arrangement. The critical wage rate in this parameterization is $\bar{\omega} = 0.83$. For any wage level w_f below this threshold, the family

Figure 4: Illustrative Example



Notes: Parameter choices are $p = 1$, $\psi = 0.4$, $\phi = 0.18$, $\kappa = 0.5$, $\sigma = 10$, $\tau_0 = 0.05$, $\alpha = 0.5$, $\nu = 0.9$.

will rely solely on the mother to take care of the children. As soon as w_f surpasses $\bar{\omega}$, the couple decides to also purchase child care on the market. This means that the mother's share in caring for children t_f declines in her wage rate, and the share of child care bought on the market τ increases.

In the top right panel, we show the couple's number of children $n^\star$ (blue line), as well as the marginal child penalty $l(\cdot)$ (solid red line) and the effective child cost $d(\cdot)$(dashed red line). When the woman's wage rate is equal to zero, the couple experiences no child penalty. All child care is done at no cost by the mother. However, since children also come with consumption costs, the couple decides to have a finite number of children $(n^\star \approx 3)$. As the maternal wage rate increases, the marginal child penalty increases in direct proportion to w_f. This results from the fact that for all wage levels $w_f < \bar{\omega}$, the mother will take care of all the work related to caring for children. The same is true for the total effective cost of children. As a result, fertility unanimously declines in this region.

Once the mother's wage rate surpasses the threshold $\bar{\omega}$, the situation changes completely. A higher maternal wage now leads to a decline in child care time for the mother and an increase in child care time bought on the market. As a result, the marginal child penalty declines again. As wages increase further, the child penalty finally converges to zero. Although the total cost of children $d(\cdot)$ increases in w_f, the couple's fertility rate rises. This results from the decline in the child penalty the mother incurs. A lower child penalty leads to an improved outside option for the mother and a better bargaining result. Since her bargaining loss declines, she is willing to agree to a greater number of children again. Finally, the total fertility rate converges to some constant value as the wage rate w_f increases further. This results from the fact that for very large wages, the couple essentially buys all child care on the market.

The lower left panel illustrates the relation between fertility and female labor supply. In the low wage regime, where women take care of all child care duties, a rise in mother's wages causes a decline in fertility and an immediate increase is female labor supply. The situation flips as the couple starts buying child care on the market. Now, a rise in the maternal wage rate leads the couple to purchase more market-based child care and to lower the mother's child care duties. The corresponding decline in the child penalty comes along with both an increase in maternal labor supply and an increase in fertility. The situation flattens out as the wage rate w_f increases further to very high levels. There, both fertility and labor supply are essentially flat lines.

The lower right panel finally shows the elasticities of maternal child care time $\tilde{t}_f$ and fertility $n^\star$ with respect to the mother's wage w_f. The statistics nicely summarize what we see in the other three panels. For low values of w_f the elasticity $\epsilon_{\tilde{t}_f,w_f}$ is equal to zero and therefore we have $\varepsilon_{n^\star,w_f} < 0$. Once w_f surpasses $\bar{\omega}$, the elasticity $\epsilon_{\tilde{t}_f,w_f}$ declines. For a while, this elasticity is still below a value of $2 + \alpha = 2.5$. Hence, the fertility decision $n^\star$ still declines in w_f, although the couples already buys child care on the market. Once the elasticity $\epsilon_{\tilde{t}_f,w_f}$ becomes greater than 2.5, however, fertility starts increasing in w_f, meaning that $\varepsilon_{n^\star,w_f}$ turns positive. For very high levels of w_f, the elasticity $\epsilon_{\tilde{t}_f,w_f}$ finally converges to a value of $-\sigma$ and $\varepsilon_{n^\star,w_f}$ converges to zero, see Proposition 2.

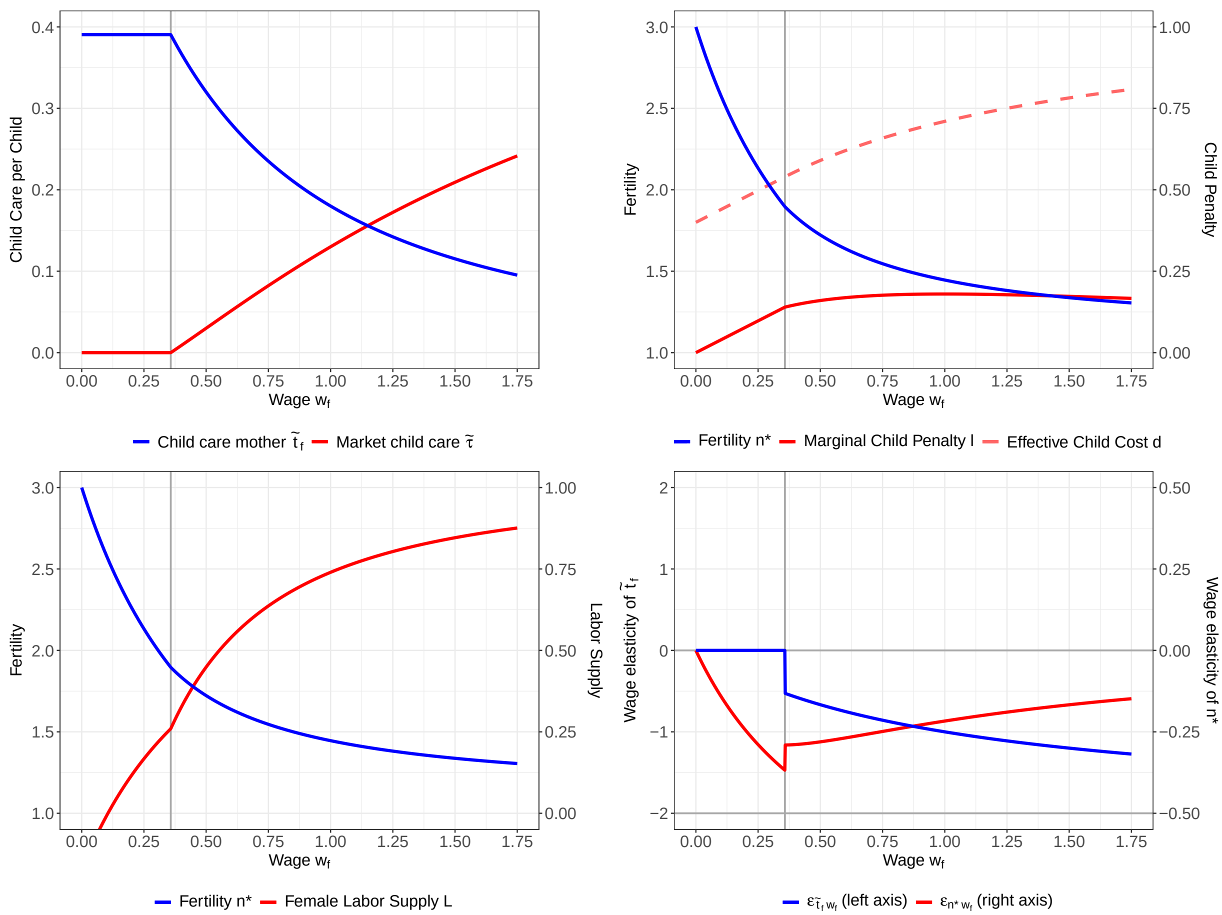
Sensitivity Analysis 1: Low elasticity of substitution ($\sigma = 2$) There are two necessary ingredients that make the non-linear dynamics of the fertility rate in our baseline scenario work. The first is a large elasticity of substitution between child care provided by the mother and child care bought on the market. As we stated in Proposition 1, whether $n^\star$ increases or decreases in w_f is majorly governed by the elasticity of maternal child care $\tilde{t}_f$ with respect to the wage rate w_f. Only if this elasticity is smaller than $-(2+\alpha)$ can there be a positive relationship between fertility and women's wages. A lower bound for $\epsilon_{\tilde{t}_f,w_f}$ is $-\sigma$. In our first sensitivity analysis, we therefore choose $\sigma = 2$ in order to ensure that $\epsilon_{\tilde{t}_f,w_f}$ can never be smaller than $-(2 + \alpha)$. The results for this configuration can be seen in Figure 5.

A low elasticity of substitution between maternal and market based child care results in a much lower movements of $\tilde{t}_f$ and τ as the maternal wage rate increases. This can be seen in the upper left panel of Figure 5. As a result, the marginal child penalty only marginally declines in w_f and it only decreases as w_f surpasses a value of 1. This small decline in the marginal child penalty is not enough to offset the overall increasing cost of children that results from higher female wages. Fertility therefore declines monotonically in w_f, see the upper left panel, and the relation between female labor force participation and fertility will always be negative (see lower two panels).

Sensitivity Analysis 2: Full commitment A second important ingredient into the nonlinear relationship between fertility and maternal wages is the loss in bargaining power mothers incur by taking care of their children. The fundamental assumption we made was that couples can not commit perfectly to sharing the full cost of children equally. If, however, couples were able to make such commitments, then the fertility decision of the couple would only depend on the total effective child costs, see (2). Such a scenario is shown in Figure 6.

While the child care arrangement and the marginal child penalty still exhibit the same dynamics as in the baseline scenario in Figure 4, there is no direct relation between the child penalty and the couple's fertility rate. Instead, it is only the total cost of children that govern the fertility decision. This has two consequences: First, as the maternal wage rate increases and the mother still takes care of the children entirely on her own, the decline in fertility is much smaller than in the baseline scenario. Second, as w_f surpasses the critical value $\bar{\omega}$ and the marginal child penalty declines quickly, there is no positive reaction in fertility. Again, the relationship between fertility and female labor force participation is unanimously negative, as can be seen from the lower two panels of Figure 6.

Figure 5: Sensitivity Analysis 1: Low Elasticity of Substitution ($\sigma = 2$)



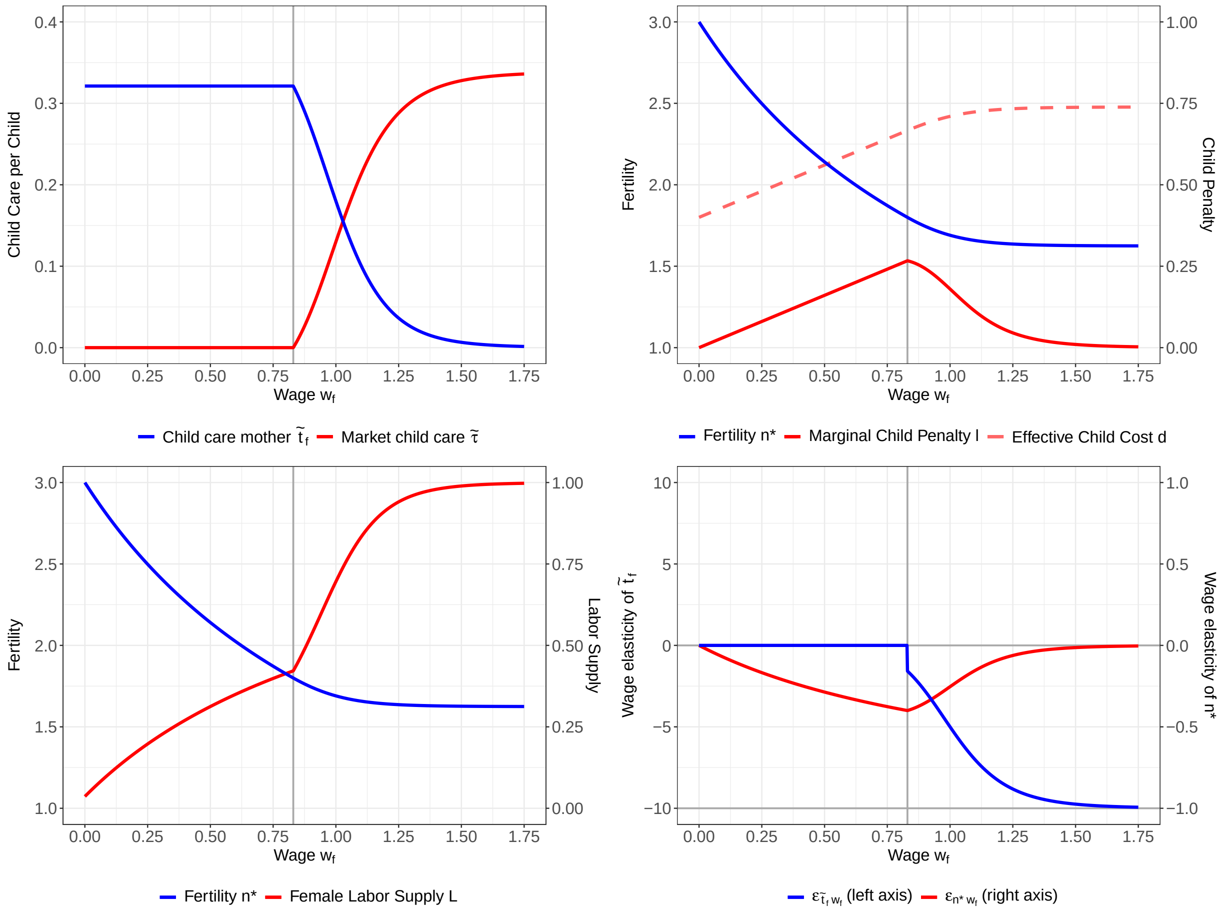
Notes: Parameter choices are $p = 1$, $\psi = 0.4$, $\phi = 0.18$, $\kappa = 0.5$, $\sigma = 2$, $\tau_0 = 0.05$, $\alpha = 0.5$, $\nu = 0.9$.

3 Empirical analysis of marginal child penalties

In this section, we provide a first proof of concept for estimating marginal child penalties using data from the German Socio-Economic Panel (SOEP). We focus on the period from 1991 to 2022, considering men and women aged 18 to 60. Using biography data, we identify individuals who had at least one child during their lifetime and calculate the age at which their first child was born. We exclude individuals whose first child was born before the age of 14 to avoid errors.

Our main variable of interest is employment status. We denote the employment state of an individual i of gender g at age j and time t by $E_{igjt} \in \{0, 1\}$. Additionally, we denote an individual's type of employment by $F_{igjt} \in \{0, 0.5, 1\}$, where 0 stands for no employment, 0.5 represents part-time work and 1 represents full-time work.

Figure 6: Alternative Scenario 2: Full Commitment



Notes: Parameter choices are $p = 1$, $\psi = 0.4$, $\phi = 0.18$, $\kappa = 0.5$, $\sigma = 10$, $\tau_0 = 0.05$, $\alpha = 0.5$, $\nu = 0.9$.

Absolute Child Penalty As a first step, we replicate the child penalty analysis of Kleven, Landais, and Søgaard (2019). Specifically, we estimate the following event-study model separately for women and men:

$$E_{igjt} = \alpha^g \times D_{igjt}^{Event} + \beta^g \times D_j^{Age} + \gamma^g \times D_t^{Year} + \epsilon_{igjt} \tag{6}$$

We include age fixed-effects D_j^{Age} and time fixed-effects D_t^{Year} to capture general age and time trends in employment. Our effect of interest is the event D_{igjt}^{Event}, which measures the time distance to the birth of the first child. Since we only include individuals who had at least one child, our identifying variation comes from the timing of births.[3] We also run the same analysis using the individual's type of employment F_{igjt} as the dependent

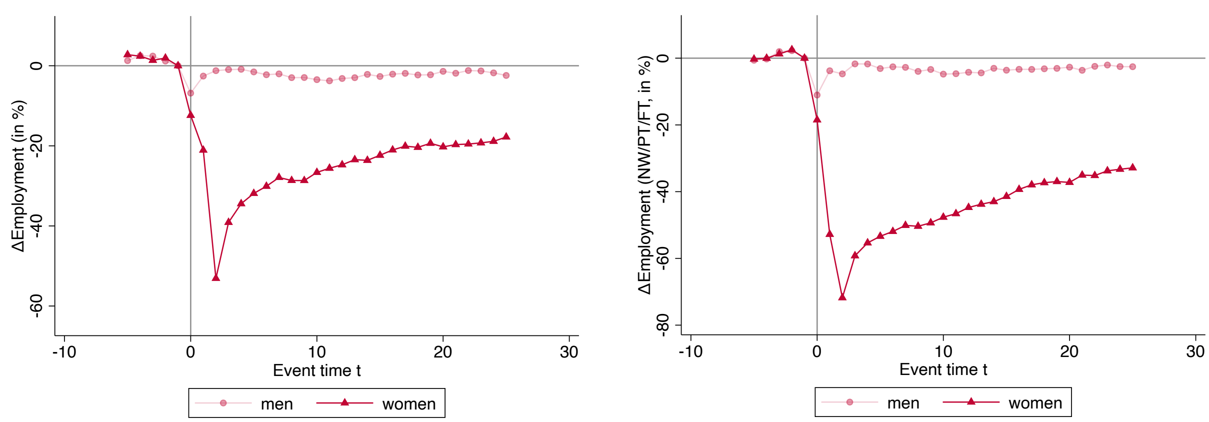
[3]Repeating our analysis with all individuals, not just those with children, yields similar results.

variable.

Figure 7 shows the absolute child penalty for both men and women. Consistent with the findings of Kleven, Landais, and Søgaard (2019) and Kleven, Landais, and Leite-Mariante (2024), men exhibit virtually no penalty from having children, neither in employment nor in their type (no work, part-time, full-time). In contrast, the child penalty for women is substantial. Upon the birth of the first child, female employment drops by around 60 percent. It recovers somewhat three years after childbirth but remains 20 to 25 percent below trend around 20 years after the birth of the first child. Although quantitatively different, a similar pattern is observed when accounting for part-time and full-time work.

Figure 7: Absolute Child Penalty for All Parents

(a) Employment (b) incl. PT and FT

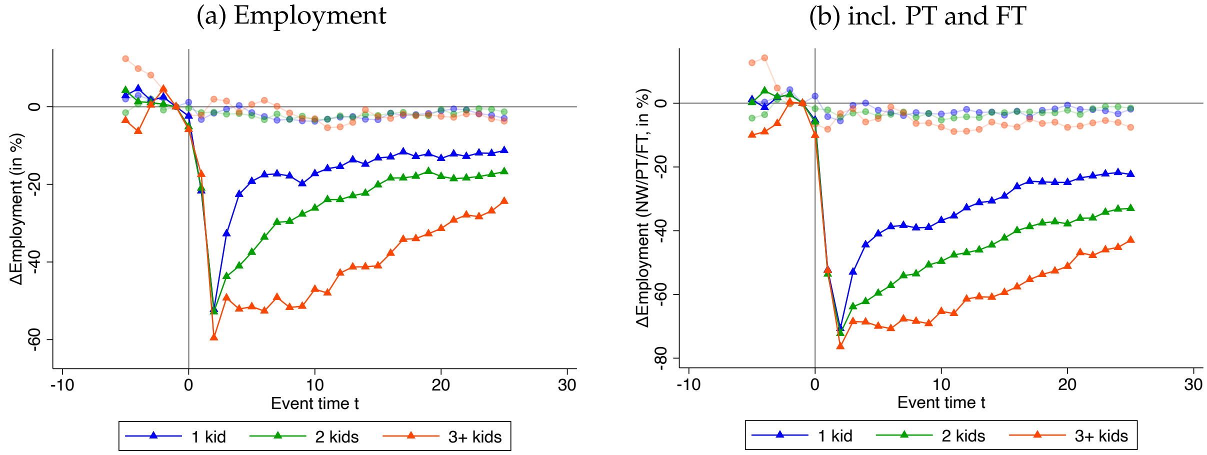


Source: Own calculations based on the German Socio-Economic Panel.

In a second step, we divide our sample into three sub-groups based on the number of children n an individual had. We then run the regression model (6) with separate coefficients $\alpha^{g,n}$ for each group to estimate the child penalty by parity. The considered event yet remains the birth of the first child. The results are shown in Figure 8. This sample split provides an initial idea of how child penalties may vary with the number of children.

Again, we find no child penalty for men, but child penalties are quite heterogeneous among women with different numbers of children. The immediate impact of having a first child is similar for all women, regardless of the total number of children. However, the speed at which women return to the labor force varies significantly. While many women with only one child return to the labor market quickly, women with three or

Figure 8: Absolute Child Penalty by Parity



Source: Own calculations based on the German Socio-Economic Panel.

more children typically remain out of the labor force for at least 10 years before gradually returning to work. A similar pattern is observed when accounting for part-time and full-time work. Yet, although mothers of only one child return to the labor force faster, they make substantial use of part-time work agreements, which leads to a larger child penalty for mothers of one child when part-time work is accounted for.

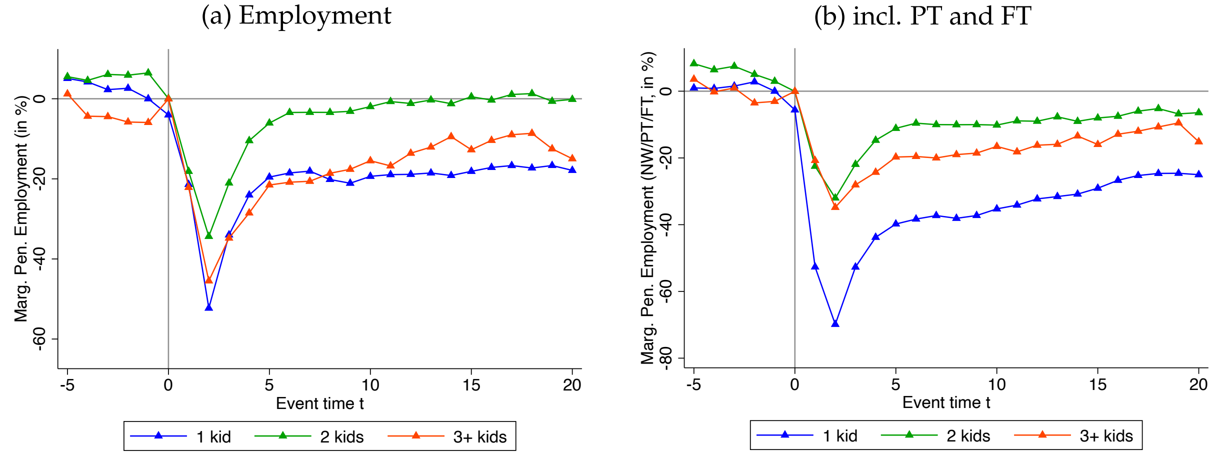
Marginal Child penalties Finally, we aim to estimate (true) marginal child penalties, i.e., the additional penalty induced by having another child. To achieve this, we estimate the following regression model:

$$E_{it} = \alpha^{g,1} \times D^{Event:n=1}_{igjt} + \mathbb{1}_{n=2} \times \alpha^{g,2} \times D^{Event:n=2}_{igjt} + \mathbb{1}_{n=3} \times \alpha^{g,3} \times D^{Event:n=3}_{igjt} + \beta^g \times D^{Age}_j + \gamma^g \times D^{Year}_t + \epsilon_{igjt}. \quad (7)$$

In this equation, the birth of each additional child is defined as a distinct event. The coefficient estimates $\alpha^{g,n}$ characterize the additional employment penalty incurred by a partner upon the arrival of the nth child. The results are shown in Figure 9.

The figure reveals distinct patterns across the number of children. Regarding the pure employment decision (employed or not), the marginal penalties are of similar magnitude for the first and third children. However, for the second child, the penalty is much smaller and, importantly, it quickly reverts to zero after approximately 5 to 6 years. This indicates that the medium to long-term employment costs (as the sum of marginal penalties) are not significantly larger for having two children compared to having one

Figure 9: Marginal Child Penalty for nth Child



Source: Own calculations based on the German Socio-Economic Panel.

child. However, third (and subsequent) children again come with a substantial additional employment penalty. When accounting for differences in part-time and full-time work, the picture changes somewhat. Here, the first child induces the highest penalty. The second and third children lead to additional declines in labor supply, but to a lesser extent. As previously discussed in Figure 8, even mothers of first children extensively use part-time work arrangements.

The effect estimates above can obviously not be interpreted as causal. There may be significant selection into the different parity groups, driven by preferences for having children, female wages, or career aspirations (Adda, Dustmann, and Stevens 2017). Nevertheless, these numbers provide a good initial assessment of the marginal cost associated with having children and may still be informative for parameterizing a quantitative version of our bargaining model.

4 Conclusion

This paper highlights the significant role of bargaining in fertility decisions, particularly through the lens of the child penalty. We show that fertility rates are influenced by bargaining frictions, which vary across different societal contexts. In traditional societies where married women typically do not work, and in gender-equal societies where childcare responsibilities are shared, these frictions are minimal. However, in societies where women aspire to work but face high child penalties, the friction is substantial, leading to lower fertility rates. This dynamic underscores the complex interplay between fertility and female labor force participation, which can evolve over time.

In reality, of course, bargaining wedges are not solely determined by women's wages but are also shaped by aspirations and social norms. The importance placed on having a career versus a family, and societal expectations regarding the roles of mothers and fathers, can be expected to significantly impact these frictions. A more detailed and quantitative version of our simple model should include such important elements of the female labor force and fertility transition. In addition, it should pay more attention to the role of modern fathers in caring for children.

In order to parameterize a quantitative version of our model, more data on marginal child penalties both in the cross-section and over time is needed. A fundamental question in this regard is the quality of available data. As shown by Kleven, Landais, and Leite-Mariante (2024), estimating overall child penalties does not necessarily require high-quality panel data. If anything, estimates from census data – that typically only features repeated cross-sections of individuals – typically lead to the same findings as estimates from high-quality panel data. One fundamental question is whether this also holds for marginal child penalties and what type of matching procedure will have to be used to derive high-quality estimates of marginal child penalties in the cross-section of countries and over time from cross-sectional data.

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Working Mothers, Social Norms, and Fertility

Appendix for Online Publication

Matthias Doepke and Fabian Kindermann

A Proofs for Propositions in Main Text

A.1 Proof of Proposition 1

The first-order condition for the optimal level of fertility is

$$2v'(n^\star) = w_f \tilde{t}_f(w_f, w_m, p) + (1+\alpha) t_f(w_f, w_m, p).$$

Total differentiation yields

$$2v''(n^\star) dn^\star = \tilde{t}_f(w_f, w_m, p) dw_f + w_f \frac{\partial \tilde{t}_f(w_f, w_m, p)}{\partial w_f} dw_f + (1+\alpha) \frac{\partial \tilde{d}(w_f, w_m, p)}{\partial w_f} dw_f.$$

We know that

$$\tilde{d}(w_f, w_m, p) = \psi + \min_{\tilde{t}_f, \tilde{\tau}} w_f \tilde{t}_f + p\tilde{\tau} \quad \text{s.t.} \quad f(\tilde{t}_f, \tilde{\tau}) = \phi.$$

The Lagrangean for the time input optimization problem reads

$$\mathcal{L} = w_f \tilde{t}_f + p\tilde{\tau} + \lambda \left[\phi - f(\tilde{t}_f, \tilde{\tau}) \right].$$

Using the envelope theorem, we get that

$$\frac{\partial \tilde{d}(w_f, w_m, p)}{\partial w_f} = \frac{\partial \mathcal{L}}{\partial w_f} = \tilde{t}_f(w_f, w_m, p).$$

We can consequently write

$$2v''(n^\star) n^\star \frac{dn^\star}{n^\star} = \left[1 + \frac{\partial \tilde{t}_f(w_f, w_m, p)}{\partial w_f} \frac{dw_f}{\tilde{t}_f(w_f, w_m, p)} + (1+\alpha) \right] w_f \tilde{t}_f(w_f, w_m, p) \frac{dw_f}{w_f},$$

which immediately leads to Proposition 1. □

A.2 Proof of Proposition 2

The elasticity of the fertility rate $n^\star$ with respect to w_f is

$$\varepsilon_{n^\star,w_f} = \frac{2+\alpha+\varepsilon_{\tilde{t}_f,w_f}}{2v''(n^\star)n^\star} \times w_f \tilde{t}_f(w_f, w_m, p).$$

Let us now assume that $w_f = 0$. In this case, the mother will obviously provide all child care on her own and no child care will be bought on the market, i.e. $\tilde{\tau}(0, w_m, p) = 0$. Hence, the cost of caring for children is zero and the only cost that remains is the monetary cost ψ. The optimal fertility decision in this case is characterized by

$$v'(n^\star(w_f = 0)) = \frac{1+\alpha}{2}\psi.$$

Consequently, we have $n^\star(w_f = 0) < \infty$, meaning that fertility is finite even in the absence of costs of child caring. Now recall that we required $\varepsilon_{\tilde{t}_f,w_f} = 0$ for all $w_f < \bar{\omega}$. Since $n^\star$ is finite, we have $v''(n^\star) < 0$ and therefore we get

$$\varepsilon_{n^\star,w_f} = \frac{2+\alpha}{2v''(n^\star)n^\star} \times w_f \tilde{t}_f(0, w_m, p) < 0 \quad \text{for all} \quad 0 < w_f < \bar{\omega}.$$

For the second part of the proposition, it is important to understand how the marginal child penalty $w_f \tilde{t}_f(w_f, w_m, p)$ evolves as w_f increases. Note that in Assumption 3 we required that

$$\lim_{w_f \to \infty} \varepsilon_{\tilde{t}_f,w_f} = -\sigma \quad \text{with} \quad \sigma > 1.$$

Consequently, there exists a wage level $\hat{w}_f$ and a value $\hat{\sigma} > 1$ such that $\varepsilon_{\tilde{t}_f,w_f} \leq -\hat{\sigma}$ for all $w_f > \hat{w}_f$. This in turn means that we can bound $\tilde{t}_f(w_f, w_m, p)$ from above as

$$\tilde{t}_f(w_f, w_m, p) \leq A w_f^{-\hat{\sigma}} \quad \text{for all} \quad w_f > \hat{w}_f \quad \text{with} \quad A = \frac{\tilde{t}_f(\hat{w}_f, w_m, p)}{\hat{w}_f^{-\hat{\sigma}}}.$$

Hence, we get with $\hat{\sigma} > 1$ that

$$\lim_{w_f \to \infty} w_f \tilde{t}_f(w_f, w_m, p) \leq \lim_{w_f \to \infty} w_f A w_f^{-\hat{\sigma}} = \lim_{w_f \to \infty} A w_f^{1-\sigma} = 0.$$

As $\tilde{t}_f(w_f, w_m, p)$ has to be non-negative, this means that

$$\lim_{w_f \to \infty} w_f \tilde{t}_f(w_f, w_m, p) = 0.$$

Finally, we arrive at

$$\lim_{w_f \to \infty} \varepsilon_{n^\star, w_f} = \frac{2 + \alpha - \sigma}{2v''\left(n^\star_{\text{long}}\right) n^\star_{\text{long}}} \times 0 = 0,$$

where $n^\star_{\text{long}}$ denotes the constant (long-run) level of fertility that the fertility rate converges to as w_f rises to infinity. □

A.3 Proof of Proposition 3

The woman's labor supply is

$$\ell(w_f, w_m, p, n) = 1 - n^\star \tilde{t}_f(w_f, w_m, p).$$

From this, we immediately get

$$\begin{aligned}
\frac{d\ell(w_f, w_m, p, n)}{dn^\star} &= -\tilde{t}_f(w_f, w_m, p) - n^\star \frac{d\tilde{t}_f(w_f, w_m, p)}{dn^\star} \\
&= -\tilde{t}_f(w_f, w_m, p) - n^\star \frac{\frac{d\tilde{t}_f(w_f, w_m, p)}{dw_f}}{\frac{dn^\star}{dw_f}} \\
&= -\tilde{t}_f(w_f, w_m, p) - \tilde{t}_f(w_f, w_m, p) \frac{\frac{d\tilde{t}_f(w_f, w_m, p)}{dw_f} \frac{w_f}{\tilde{t}_f(w_f, w_m, p)}}{\frac{dn^\star}{dw_f} \frac{w_f}{n^\star}} \\
&= -\tilde{t}_f(w_f, w_m, p) \left[1 + \frac{\varepsilon_{\tilde{t}_f, w_f}}{\varepsilon_{n^\star, w_f}}\right].
\end{aligned}$$

From this, we immediately obtain

$$\varepsilon_{\ell, n^\star} = \frac{d\ell(w_f, w_m, p, n)}{dn^\star} \frac{n^*}{\ell(w_f, w_m, p, n)} = -\frac{n^\star \tilde{t}_f(w_f, w_m, p)}{\ell(w_f, w_m, p, n)} \times \left[1 + \frac{\varepsilon_{\tilde{t}_f, w_f}}{\varepsilon_{n^\star, w_f}}\right].$$

□